Marshall@Colloq: L^p norms of eigenfunctions on locally symmetric spaces

January-06-14 3:09 PM

M^n - cmpct Riemannian manifold $\Delta\psi = \lambda^2\psi$

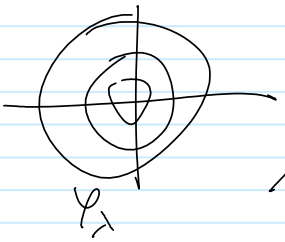
Will like to study ψ as $\lambda \to \infty$.

Take $\|\psi\|_2 = 1$, try to bound $\|\psi\|_p$ as $2 < p \leq \infty$.

$p = \infty$ Avacumsvic-Levitan: $\|\psi\|_\infty \ll \lambda^{(n-1)/2}$

$\ll$ means less than a const. times

Example in $\mathbb{R}^2$, realizes the bound.

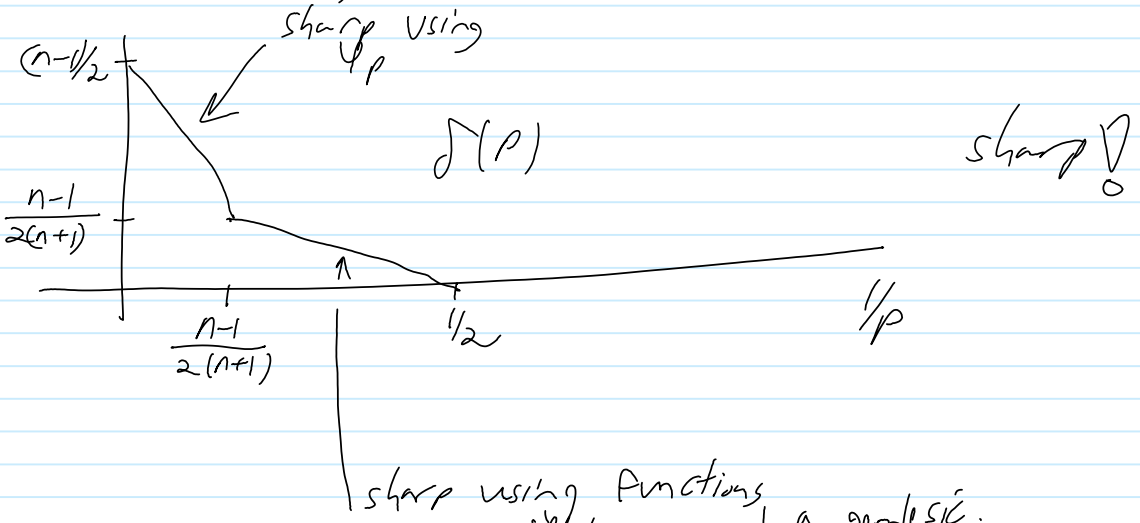


spherically symmetric

$\Delta\varphi_\lambda = \lambda^2\varphi_\lambda$

φ_λ

$p < \infty$ Thm (Sogge) $\|\psi\|_p \ll \lambda^{\delta(p)}$



Also have a complete answer as $N \subset M$ for

$\|\psi|_N\|_p$

$\mathbb{H}^2 \cong SL_2(\mathbb{R})/SO(2)$

$\mathbb{H}^3 \cong SL_3(\mathbb{R})/SO(3)$

R = Ring of invt. differential operators.

For $\mathbb{H}^3$, $R \cong \mathbb{C}[\Delta, D]$

Δ: Laplacian, deg 2; D: $\frac{1}{i}$, deg 3